LITERATURE CITED

Bungay, H. R., III, W. J. Whalen, and W. M. Sanders III, "Microprobe Techniques for Determining Diffusivities and Respiration Rates in Microbial Slime Systems," *Biotechnol. Bioengr.*, **11**, 765 (1969).

Bungay, H. R., III, W. M. Sanders III, and W. J. Whalen, "Oxygen Transfer at the Microscopic Level," Abstracts of Papers, Am. Chem. Soc. 160th National Meeting, Chicago (1970).

Danckwerts, P. V., "Significance of Liquid-Film Coefficient in Gas Absorption," *Ind. Eng. Chem.*, **43**, 1460 (1951).

Marchello, J. M., and H. L. Toor, "A Mixing Model for Transfer Near a Boundary," *Ind. Eng. Chem. Fundamentals*, **2**, 8 (1963).

Ruckenstein, E., "On Turbulent Transfer Near a Liquid-Fluid Interface," *Chem. Eng. J.*, **2**, 1 (1971).

Sanders, W. M., III, H. R. Bungay, III, and W. J. Whalen, "Oxygen Microprobe Studies of Microbial Slime Films," *Chem. Eng. Progr. Symp. Ser. No. 107*, **67**, 69 (1971).

Toor, H. L., and J. M. Marchello, "Film-Penetration Model for Mass Transfer and Heat Transfer," *AIChE J.*, **4**, 97 (1958).

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Empirical Expressions for the Shear Stress in Turbulent Flow in Commercial Pipe

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The method of Churchill and Usagi (1972) is used herein to develop expressions for the shear stress on the wall in turbulent flow in commercial pipe at all Reynolds numbers and all roughness ratios. In this application the objective is to derive an equation for interpolation between the empirical correlations which have previously been developed for smooth pipe and for fully developed turbulence at very large Reynolds number in rough pipe. The method itself is straightforward, but the simplicity and accuracy of the final expression depends not only on the choice of the limiting correlations but also on the form in which they are arranged. Several such alternatives are examined.

Nikuradse (1932) developed the following correlations for his own precise experimental measurements of the pressure drop in turbulent flow in smooth pipes and in fully developed turbulent flow in pipes with uniform, artificial roughness:

$$\frac{u}{\sqrt{\tau/\rho}} = 2.46 \ln\left(\frac{D}{\nu}\sqrt{\frac{\tau}{\rho}}\right) + 0.30 \qquad (1)$$

and

$$\frac{u}{\sqrt{\tau/\rho}} = 2.46 \ln\left(\frac{D}{\epsilon}\right) + 3.22 \qquad (2)$$

The roughness of commercial pipes is not uniform and hence is not uniquely defined. Colebrook (1938-1939) proposed that this roughness be defined arbitrarily as the effective value such that the experimental measurements of pressure drop at very large Reynolds number fit Equation (2). He further observed that when expressed in this form the experimental data for commercial pipes vary uniformly between the limiting behavior represented by Equations (1) and (2). This is a necessary condition for application of the method of Churchill and Usagi.

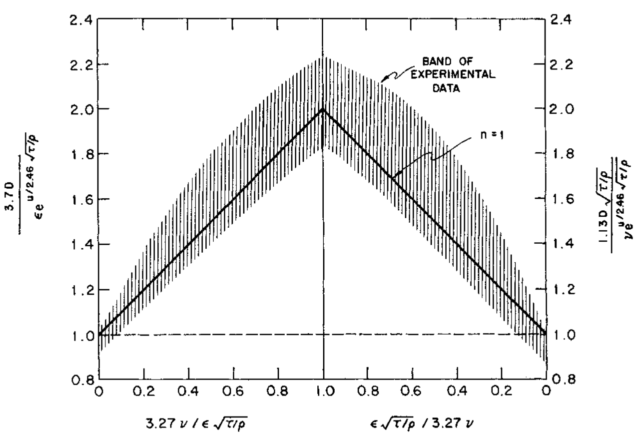

Fig. 1. Graphical construction of correlation.

Nikuradse rearranged Equations (1) and (2) in the form

$$\frac{u}{\sqrt{\tau/\rho}} - 2.46 \ln\left(\frac{D}{\epsilon}\right) = 2.46 \ln\left(\frac{\epsilon}{\nu}\sqrt{\frac{\tau}{\rho}}\right) + 0.30 \tag{3}$$

and

$$\frac{u}{\sqrt{\tau/\rho}} - 2.46 \ln\left(\frac{D}{\epsilon}\right) = 3.22 \tag{4}$$

in order to develop a general graphical correlation for uniformly roughened and smooth pipe. The corresponding trial expression of Churchill and Usagi then takes the form

$$\frac{3.22}{\dfrac{u}{\sqrt{\tau/\rho}} - 2.46 \ln\left(\dfrac{D}{\epsilon}\right)} = \left[1 + \left[\frac{3.22}{2.46 \ln\left(\dfrac{1.13\,\epsilon}{\nu}\sqrt{\tau/\rho}\right)}\right]^{n}\right]^{1/n} \tag{5}$$

However, the additive combination of variables on the left side of Equations (3) and (4) is quite arbitrary and Equation (5) does not prove to be the best form for interpolation.

Equations (1) and (2) can be combined directly in a trial expression as follows without rearrangement except for inclusion of the additive terms in the logarithmic terms:

$$\frac{2.46 \ln\left(\dfrac{3.7D}{\epsilon}\right)}{u/\sqrt{\tau/\rho}} = \left[1 + \left[\frac{\ln\dfrac{3.7D}{\epsilon}}{\ln\left(\dfrac{1.13D}{\nu}\sqrt{\dfrac{\tau}{\rho}}\right)}\right]^{n}\right]^{1/n} \tag{6}$$

Equation (6) is less arbitrary and somewhat more successful for interpolation than Equation (5).

A much better correlation is obtained if both Equations (1) and (2) are divided by 2.46 and raised to the e-th power:

$$e^{u/2.46\sqrt{\tau/\rho}} = \frac{1.13D}{\nu}\sqrt{\frac{\tau}{\rho}} \tag{7}$$

$$e^{u/2.46\sqrt{\tau/\rho}} = \frac{3.7D}{\epsilon} \tag{8}$$

The trial expression is then

$$\frac{3.7D}{\epsilon\, e^{u/2.46\sqrt{\tau/\rho}}} = \left[1 + \left(\frac{3.27\nu}{\epsilon}\sqrt{\frac{\rho}{\tau}}\right)^{n}\right]^{1/n} \tag{9}$$

This form is tested in Figure 1 in which the band of experimental data correlated graphically by Rouse and Howe (1953) is represented by the shaded region. The straight lines representing $n = 1$ fall within this band. Hence the final correlation can be written

$$e^{-u/2.46\sqrt{\tau/\rho}} = \frac{\epsilon}{3.7D} + \frac{\nu}{1.13D}\sqrt{\frac{\rho}{\tau}} \tag{10}$$

This expression is identical to that derived by Colebrook by heuristic reasoning. (He asserted without rationalization that $\epsilon/3.7D$ and $\nu\sqrt{\rho/\tau}/1.13D$ must be additive.) Equation (10) is explicit in u but is implicit in τ.

Colebrook noted that the expression

$$\frac{u}{\sqrt{\tau/\rho}} = 2.21 \ln\left(\frac{Du}{7\nu}\right) \tag{11}$$

provides almost as good a representation as Equation (1) for the experimental data for turbulent flow in smooth pipes and has the advantage of being explicit in τ. However, he did not use Equation (11) to construct a general correlation as an alternative to Equation (10). Dividing Equation (11) through by 2.46, raising both sides to the e-th power, and combining with Equation (8) gives the trial expression

$$\frac{3.7D}{\epsilon\, e^{u/2.46\sqrt{\tau/\rho}}} = \left[1 + \left(\frac{21.3D}{\epsilon}\left(\frac{\nu}{Du}\right)^{0.9}\right)^{n}\right]^{1/n} \tag{12}$$

The best value of n is necessarily the same as for Equation (9), yielding

$$e^{-u/2.46\sqrt{\tau/\rho}} = \frac{\epsilon}{3.7D} + \left(\frac{7\nu}{Du}\right)^{0.9} \tag{13}$$

Equation (13) is implicit in u but explicit in τ and hence is more convenient that Equation (10) for process calculations. Equations (10) and (13) differ only in the second term on the right and give essentially the same values. The choice depends primarily on whether the flow rate or the pressure drop is specified.

Figure 1 is a plot of the deviations from the two limiting solutions in exponential form in arithmetic coordinates. Hence, the scatter of the data, the deviations from Equation (10), and the magnitude of the deviations in shear stress are all exaggerated. The experimental data deviate to an approximate maximum of 20% above and 10% below Equation (10). A deviation of 20% in Figure 1 is equivalent to a deviation of less than 6% in the shear stress itself. Since some of the deviation in Figure 1 is undoubtedly due to experimental error rather than to failure of the equation to represent the physical behavior, both Equations (10) and (13) are probably reliable to within even narrower limits. They are therefore recommended as more convenient and accurate for design calculations than the conventional plots of the friction factor.

NOTATION

D = diameter of pipe, m
u = mean velocity in pipe, m/s
ϵ = roughness, m
ν = kinematic viscosity, m^2/s
ρ = density, kg/m^3
τ = shear stress on wall, N/m^2

LITERATURE CITED

Churchill, S. W., and R. Usagi, "A General Expression for the Correlation of Rates of Transfer and Other Phenomena," *AIChE J.*, 18, 1121 (1972).

Colebrook, C. F., "Turbulent Flow in Pipes with Particular Reference to the Transition Between the Smooth and Rough Pipe Laws," *J. Inst. Civil Eng.*, 11, 133 (1938-1939).

Nikuradse, J., "Strömungsgesetze in rauhen Rohren," *V.D.I. Forschungsheft*, Heft 361, 1 (1933); English Trans. *Petro-Leum Eng.*, No. 3, 164 (1940), No. 5, 75 (1940), No. 6, 124 (1940), No. 7, 38 (1940), No. 8, 83 (1940).

Rouse, H., and J. W. Howe, *Basic Mechanics of Fluids*, p. 143, Wiley, N. Y. (1953).

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